

## Part II: Understanding Monetary Policy

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### 1 Inflation, money growth and interest rates

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### 2 Discretion and the conduct of policy: The dynamic inconsistency of low-inflation monetary policy

From “Principles of Macroeconomics” you know that money growth is the key determinant of inflation. To understand what causes high inflation, it’s thus important to understand what causes high money growth. There can be 2 reasons: seignorage (when government collects revenue from money creation) and the output-inflation tradeoff. For the major industrialized countries, the latter is the leading candidate.

We usually think (and it’s indeed true) that there is no permanent tradeoff between inflation and output in the long run. Indeed, if you imagine 2 countries with different *levels* of inflation for similar other characteristics, there is no reason for output to behave differentially under the low-inflation policy then under the high-inflation one. However, Kydland and Prescott (1997) show that the inability of policymakers to commit themselves to such low-inflation policy can give rise to excessive inflation despite the absence of a long-run trade-off.

The logic is simple: While the marginal cost of additional inflation is low, policymakers will run expansionary monetary policy to push output temporarily up. But as the public is rational, they will expect that move, and they will not in fact expect low inflation. The end result would be high inflation without any increase in output.

## 2.1 Assumptions

The aggregate supply is given by the Lucas supply curve as:

$$y = \bar{y} + b(\pi - \pi^E) \tag{1}$$

where  $y$  is the log of real output, and  $\bar{y}$  is the log of it's natural rate.  $\pi$  is inflation, and  $\pi^E$  is it's expected value next period. The model assumes that  $\bar{y} < y^*$  – the socially optimal level of production. There can be several reason for that, for example – imperfect competition that even in the long run leads to underproduction. In this sense,  $\bar{y}$  can be viewed as being a Nash equilibrium.

A policymaker cares about both production gap and excess inflation, and thus minimized the following loss function  $L$ :

$$L = (y - y^*)^2 + a(\pi - \pi^*)^2 \tag{2}$$

where  $\pi^* > 0$  is the socially optimal level of inflation and  $a > 0$  is a parameter that reflects the relative importance of output and inflation in socoail welfare. Note that this loss function reflects not just the policymaker's preferences, but also the representative individual's.

## 2.2 The solution

### 2.2.1 With a binding commitment.

Suppose that a policymaker makes a binding commitment about what inflation will be in the future before it is actually determined. A commitment means that the policymaker now *has* to run policy in a way that it leads to the promised level of inflation that the public expects. As  $\pi = \pi^E$ , it follows from the supply curve that  $y = \bar{y}$ . Thus, the policymaker's minimization problem becomes one-dimensional – finding  $\pi = \operatorname{argmin} [(\bar{y} - y^*)^2 + (\pi - \pi^*)^2]$ . The solution is simply  $\pi^{**} = \pi^*$ , resulting in  $L^{**} = (\bar{y} - y^*)^2$ .

### 2.2.2 Without a binding commitment.

Now assume that the policymaker takes expectations as given, and has no obligations to actually fulfil them. In fact, the policymaker uses discretion and can choose any policy that maximizes social welfare (i.e. minimizes  $L$ ). There are many reasons why the expectations might be formed in advance, for example, if the policymaker ran credible monetary policy before.

If we take the supply curve and plug it into the loss function, the problem the policymaker faces takes the following form:

$$\min L = [\bar{y} + b(\pi - \pi^E) - y^*]^2 + a(\pi - \pi^*)^2 \quad (3)$$

which needs to be maximized wrt.  $\pi$ . The FOC (after we divide by 2) is

$$b[\bar{y} + b(\pi - \pi^E) - y^*] + a(\pi - \pi^*) = 0 \quad (4)$$

If we now solve it for  $\pi$ , we should get that

$$\pi = \frac{b^2\pi^2 + a\pi^* + b(y^* - \bar{y})}{a + b^2} \quad (5)$$

$$= \pi^* + \frac{b}{a + b^2}(y^* - \bar{y}) + \frac{b^2}{a + b^2}(\pi^e - \pi^*) \quad (6)$$

In the equilibrium (“steady state”), inflation  $\pi^{**}$  should be equal to expected inflation  $\pi^{**} = \pi^E$ .

If we now solve the previous equation for  $\pi^E$ , we would get:

$$\pi^{**} = \pi^E = \pi^* + \frac{b}{a}(y^* - \bar{y}) \quad (7)$$

As by assumption,  $y^* > \bar{y}$ , the equilibrium level of inflation  $\pi^{**}$  would exceed the socially optimal level  $\pi^*$ , leading to a suboptimal  $L^{**} = (1 + \frac{b^2}{a})(\bar{y} - y^*)^2$ .

Is there any gain in the output? No. Indeed, as equilibrium inflation equals expected inflation,  $\pi = \pi^{**} = \pi^E$ , the Lucas supply curve leads to  $y = \bar{y}$ . In short, all the policymaker’s “discretion” does is to increase inflation without affecting output.

### 2.2.3 Discussion.

The ability to choose inflation after the expectations are formed makes the policymaker worse off. The problem is that for a policymaker, announcing inflation, and then actually producing it is “decanically inconsistency,” as it will always be beneficial for a policymaker to depart from his promises. However, the public know this, and takes into account. Thus, the *knowledge* that the policymaker has discretion, rather than the discretion itself, that is the source of the problem. So, even if the policymaker understands all the game and is willing to commit, he is not able to

do that as the public would not believe him.

### 2.3 Addressing the dynamic inconsistency problem through delegation

One of the ways, widely used in real time, to overcome dynamic inconsistency is by delegating monetary policy conduct to an authority that is much more concerned with inflation than the rest of the public. The idea, due to Rogoff (1985), is simple: inflation – and hence expected inflation – is lower when monetary policy is controlled by someone who is known to be especially averse to inflation. That individual’s objective function would be:

$$L = (y - y^*)^2 + \tilde{a}(\pi - \pi^*)^2 \quad (8)$$

where  $\tilde{a} \gg a$ . For a person with these preferences, the choice of optimal inflation would be:

$$\pi = \pi^* + \frac{b}{\tilde{a} + b^2}(y^* - \bar{y}) + \frac{b^2}{\tilde{a} + b^2}(\pi^e - \pi^*) \quad (9)$$

If we again solve this for a steady state equilibrium, we would see that because policymaker puts more weight on inflation than before, he or she chooses a lower value of inflation for a given level of expected inflation; in addition, his response function is flatter. As before, in equilibrium, we would get:

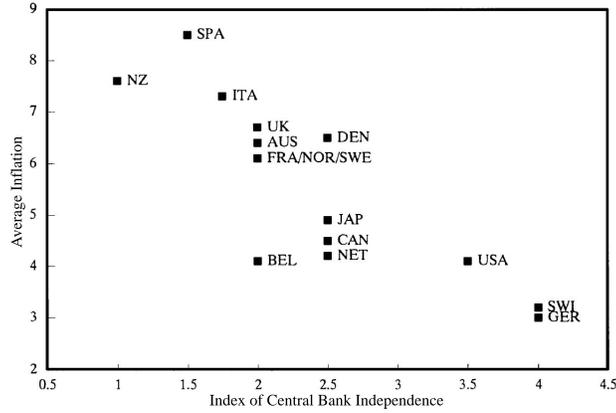
$$\tilde{\pi}^{**} = \pi^E = \pi^* + \frac{b}{\tilde{a}}(y^* - \bar{y}) \quad (10)$$

which is both actual and expected inflation, and output gap will be zero with  $y = \bar{y}$ . Note that  $\tilde{\pi}^{**}$  is closer to  $\pi^*$  – the socially optimal level of inflation – than  $\pi^{**}$ , and as  $y = \bar{y}$  in both case, delegating leads to higher social welfare with  $\tilde{L}^{***} < L^{***}$ .

Intuitively, when monetary policy is controlled by someone who cares strongly about inflation, the public realizes that the policymaker has little desire to pursue expansionary policy; the result is that expected inflation is low. Thus, choosing policymakers with a stronger dislike of inflation produces a better performance in terms of average inflation (but a worse one in terms of stabilizing output). As a result, there is some optimal level of “conservatism” for central banks.

The idea that the society can address the dynamic inconsistency problem by letting individuals who dislike inflation control monetary policy appears realistic. In many countries, monetary policy is determined by independent central banks rather than by the central govern-

ment. Some studies have (e.g. Alesina and Summers, JMCB 1988) found that more independent central banks usually manage to keep inflation at lower levels:



Note that this finding does not mean that central bank's independence is the only source of low inflation. Posen (FIE 1993) observes that countries whose citizens are particularly averse of inflation are likely to try to insulate their central banks from political pressure.

## 2.4 How important are fluctuations in consumption?

Lucas (BB 1987) has estimated the potential welfare gain from output stabilization (formally: consumption around its mean). This gain appears to be extremely small. His argument is straightforward. Suppose utility is CRRA:  $U(C) = \frac{C^{1-\sigma}-1}{1-\sigma}$  and  $\bar{C}$  is the mean value of consumption  $C$ . Since  $U''(C) = \sigma C^{-\sigma-1}$ , if we use a second-order Taylor's approximation of  $U(C)$  around its mean and take the expectation, we would get:

$$E[U(C)] \approx \frac{\bar{C}^{1-\sigma} - 1}{1 - \sigma} - \frac{\sigma}{2} \bar{C}^{-\sigma-1} E[(C - \bar{C})^2] \quad (11)$$

where  $E[(C - \bar{C})^2] = \epsilon^2$  is the variance of consumption. Elimination variability of consumption would raise expected utility by  $\Delta U(C) = \frac{\sigma}{2} \bar{C}^{-\sigma-1} \epsilon^2$ .

Let's estimate by how much consumption should increase over its average value  $\bar{C}$  to result in this increase in utility. From the Taylor's decomposition,  $\Delta U(C) = U'(\bar{C})\Delta C = \bar{C}^{-\sigma} \Delta C$ . Thus,  $\Delta C = \frac{\Delta U(C)}{\bar{C}^{-\sigma}} = \frac{\sigma}{2} \bar{C}^{-1} \epsilon^2$ . If now we express this as a fraction of average consumption  $\bar{C}$ , this would be equal to  $\frac{\sigma}{2} (\epsilon/\bar{C})^2$ .

Lucas estimates the standard deviation of consumption due to short-run fluctuations to be 1.5 percent of its mean ( $\epsilon/\bar{C} = 0.015$ ), the coefficient of relative risk aversion  $\sigma \leq 5$ , leading to

an optimistic figure for the maximum possible welfare gain from more successful stabilization policy to be 0.0006 or 0.06 percent – a very small amount.

### 3 Policy rules and the conduct of policy: The Taylor rule

#### 3.1 Interest-rate rules

Central banks for the most part conduct monetary policy not by trying to achieve some target growth rate for the money stock, but by adjusting the short term nominal interest rate in response to various disturbances.

In contrast to money-stock rules, interest rate rules *cannot* be passive. For example, suppose that the CB keeps the nominal interest rate constant, and there is a disturbance to aggregate demand that pushes output above the potential, causing inflation to rise (recall the AS-AD model from “Principles of macroeconomic.”) With the nominal interest rate fixed, this lowers the real interest rate, further pushing production upwards, and so on.

Taylor, 1993 proposed a simple interest rate rule that is linear in inflation  $\pi$  and the output gap  $\hat{y}$  (the percentage departure of output from its natural rate) and describes well the behavior of the Fed. The monetary policy rule postulated to be followed by the Fed is:

$$i_t^* = \pi_t + \delta(\pi_t - \pi^*) + \gamma\hat{y}_t + r^* \quad (12)$$

where  $i_t^*$  is the short-term nominal interest target,  $\pi^*$  is the target level of inflation and  $r^*$  is the equilibrium level of the real interest rate (both usually considered to be 2%), and  $\delta, \gamma > 0$ . The parameters  $r^*$  and  $\pi^*$  can be combined into one constant term, which leads to the following equation:

$$i_t^* = \alpha + (1 + \delta)\pi_t + \gamma\hat{y}_t \quad (13)$$

The condition  $\delta > 0$ , known as the Taylor principle, states that, when inflation rises above target, the Fed raises the nominal interest rate by more than point-for-point, so that the real interest rate rises. This has been emphasized by Taylor as the crucial condition for economic stability.

To understand why this is the case, let’s build a simple text-book macroeconomic model that

consists of an IS curve:

$$\hat{y}_t = -\beta(r_t - r^*) + \epsilon_t^D \quad (14)$$

and a Phillips curve:

$$\pi_t = \pi_{t-1} + \alpha\hat{y}_t + \epsilon_t^S \quad (15)$$

Now consider the following thought experiment. Start with inflation equal to its target level and the output gap equal to zero. Now suppose there is a positive shock to inflation. If the Taylor principle is satisfied, the Fed would raise the nominal interest rate  $i$  more than point-for-point, increasing the real interest rate  $r = i - \pi$ . The increase in the real interest rate will lead to a negative output gap by the IS curve and, in turn, to a decrease in inflation by the Phillips curve. The process will continue until inflation returned to its original, target, level. Since there is no long-run effect of the shock, inflation is stationary if the Taylor principle is satisfied.

Now consider the same shock to inflation if the Taylor principle is not satisfied. Suppose that  $\delta = 0$ . In this case, the Fed would raise the nominal interest rate in exactly point-for-point, leaving the real interest rate unchanged. With an unchanged real interest rate, the output gap in would stay at 0 and inflation in would not be brought down. Since the effect of the shock never dissipates, inflation has a unit root if the Taylor principle is not satisfied.

There is evidence that rather than making an instantaneous adjustment of the Federal Funds Rate  $i$  towards its target level  $i^*$ , the Fed tends to smooth changes in the interest rate. Following Clarida, Gali, and Gertler (EER 1998), this is usually modeled as  $i_t = (1 - \rho)i_t^* + \rho i_{t-1}$ , where  $\rho$  is the degree of smoothing: The more instantaneous the response to the shocks, the more  $\rho$  tends to zero. This transforms the Taylor rule into the following form:

$$i_t = \rho i_{t-1} + (1 - \rho)\{\alpha + (1 + \delta)\pi_t + \gamma\hat{y}_t\} \quad (16)$$

## 3.2 Deriving the Taylor rule

### 3.2.1 Assumptions

Let's try to show that the optimal policy that maximizes social welfare  $W$  (or alternatively, minimize the loss function  $L$ )

$$L = E[(y - y^*)^2] + \lambda E[\pi^2] \quad (17)$$

can indeed be described by a Taylor-type rule (this is based on Ball, IF 1999). To do this we consider an economy that is described as before by 2 equations: the aggregate demand (an IS curve) and the aggregate supply (an accelerationist Phillips curve). Unlike the previous case, now each equation is assumed to work with a lag. The IS curve takes the following form:

$$\hat{y}_t = -\beta(r_{t-1} - r^*) + \rho\hat{y}_{t-1} + \epsilon_t^D \quad (18)$$

and the Phillips curve is:

$$\pi_t = \pi_{t-1} + \alpha\hat{y}_{t-1} + \epsilon_t^S \quad (19)$$

Thus, if a policymaker adjusts  $r_t$ , it will only be able to affect output 1 period ahead  $\hat{y}_{t+1}$  (it's evident from the IS curve). Moreover, it will take one more period for inflation to experience any effect through the Phillips curve. Thus,  $r_t$  can only affect  $\pi_{t+2}$ .

### 3.2.2 The solution

First note that a policymaker's choice of  $r_t$  uniquely determined the expected value of  $\hat{y}_{t+1} = -\beta(r_t - r^*) + \rho\hat{y}_t + \epsilon_{t+1}^D$  as  $E[\hat{y}_{t+1}] = -\beta(r_t - r^*) + \rho\hat{y}_t$  (note that the expected value of  $\epsilon_{t+1}^D$  at time  $t$  is zero). Thus, we can think of monetary policy conduct as of setting  $E[\hat{y}_{t+1}]$  instead of  $r_t$ .

What does inflation 2 periods ahead,  $\pi_{t+2}$ , depends on? It depends on all the variables that characterize the economy at time  $t + 1$ . Those are  $E[\hat{y}_{t+1}]$  (which is a function of  $r_t$  and thus is determined by the policymaker) and  $E[\pi_{t+1}]$  (which does not depend on  $r_t$  as it works with a 2-period lag). Thus, we can think of inflation at  $t + 2$  as  $E[\pi_{t+2}] = F(E[\hat{y}_{t+1}(r_t)], E[\pi_{t+1}])$ . Therefore, to be able to achieve the desired level of inflation, the policymaker has to take into account  $E[\pi_{t+1}]$ , in other words, his policy response should be a function of  $E[\pi_{t+1}]$ :  $E[\hat{y}_{t+1}(r_t)] = \tilde{F}(E[\pi_{t+1}])$ . Near the equilibrium, it can be linearized, leading to:

$$E[\hat{y}_{t+1}(r_t)] = c_0 - c_1 E[\pi_{t+1}] \quad (20)$$

Our goal is finding optimal  $c_0$  and  $c_1$ .

Let's start with  $c_0$  as it is easy to find. As there is no long-run trade-off between inflation and the output gap, the Phillips curve implies that when inflation is on it's target ( $E[\pi_{t+1}] = 0$ ) the central bank sets  $E[\hat{y}_{t+1}(r_t)] = 0$ . Thus,  $c_0 = 0$ .

Before we start looking for the optimal value of  $c_1$ , note that we have already shown that the policymaker's optimal response should take the form of the Taylor rule. Indeed, let's take the "optimality" equation above, and plug in both the IS and Phillips curves, taken one period ahead:

$$-\beta(r_t - r^*) + \rho\hat{y}_t = -c_1(\pi_t + \alpha\hat{y}_t) \quad (21)$$

If we now express  $r_t$  as a function of inflation and the output gap, we would get:

$$r_t = r^* + \frac{c_1}{\beta}\pi_t + \left(\frac{\rho + c_1\alpha}{\beta}\right)\hat{y}_t \quad (22)$$

### 3.3 Issues in estimating the Taylor rule

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### 3.4 Issues in the design of interest rate rules

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### 3.5 The zero lower bound on the nominal interest rate

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